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BER Analysis of Receive Diversity Using Multiple Antenna System and MRC

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Abstract

In this work, the performance analysis of a wireless communication is done by modeling a wireless communication system and analysing its bit error rate (BER). The BER of any communication system is helpful for it performance analysis. So, the consequences of multiple antenna system on BER at receiver end is analysed and maximal ratio combining (MRC) is also estimated between signal and noise. Therefore, the BER is attempted to reduce in the wireless communication system which can affect to receive the pure signal in the receiver.

Keywords: *BER; deep fading; beam forming.*

1. Introduction

The performance of wireless communication system is compromised due to various fading channel in the multipath propagation of a message signal. The performance is measured in terms of BER. There are various methods which can be employed to reduce the BER of a wireless communication system, one of them is diversity. In this work, deep fading condition is analyzed and diversity at the receiving side is introduced to mitigate the bad performance due to deep fading event. In a wireless communication system, we cannot always get the line of sight (LOS). There are many scattered reflected paths through which the waves travel. The objects which scatter the wireless signal are called scatterers (e. g. trees, cars, buildings etc). So, such environment at receiver is called multipath environment. Some wireless signals are scattered into different paths and phase delays due to path losses. So, path difference leads to phase difference.

At the receiver, this multiple scattered signals add constructively or destructively causing constructive or destructive interference. When destructive interference occurs, the signal power is significantly diminished and this phenomenon is called fading. At the same time, maximal ratio combining (MRC) between signal and noise is also estimated.

2. Modeling of Multipath Wireless Communication System

In the wireless channel, the resultant signal at receiver side is given by the convolution of message signal x(t) and and h(t) the impulse response of the channel as shown in Figure 1.

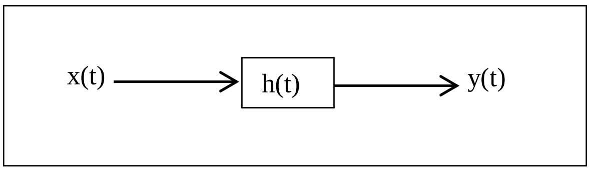

Figure1. Communication system model

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$y(t) = x(t) \otimes h(t)$

Let us consider a multipath propagation system as shown in Figure 2.

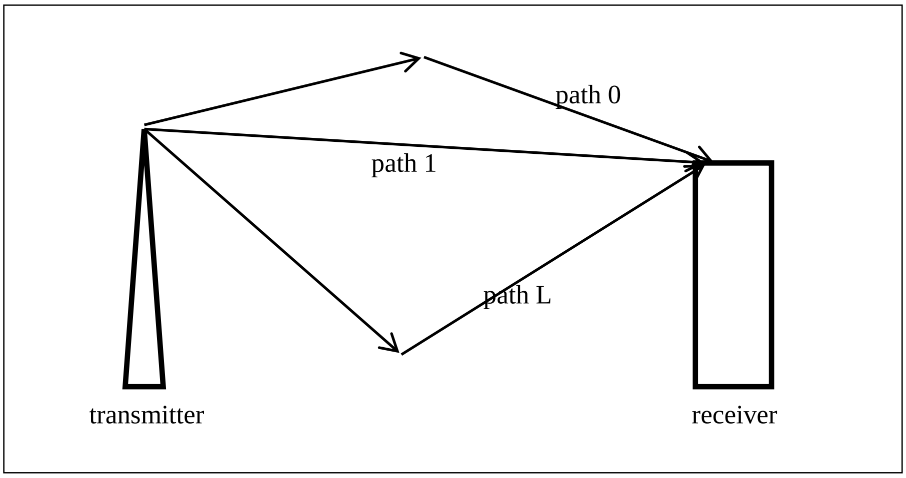


Figure 2. Multipath Wireless Communication system.

This system consists of L paths, the impulse response of each path

For 0^{th} path the channel impulse response is

$h_0(t)=a_0\ \delta(t-t_0)$

1^{st} path

$h1(t)=a1\ \delta(t-t_1)$

2^{nd} path

$h2(t)= a_2\ \delta(t-t_2)$

Lth path

$hL-1(t) = aL-1\ \delta(t-tL-1)$

$h(t)= a_0\delta(t-t_0) + a1\delta(t-t_1)+a2\ \delta(t-t_2)+\ldots\ldots\ldots\ldots\ldots+aL-1\ \delta(t-tL-1)$

$\mathbf{h(t)} = \sum_{i=0}^{L-1} ai\,\delta(t-ti)$

Wireless signals can be denoted as: (At the receiver side)

$s(t) = Re\{sb(t)e\hat{\ }j2\pi f_0t\}$

At the receiver's side

$y(t) = s(t) \otimes h(t)$

$y0(t) = Re\{a_0sb(t-t_0)e\hat{\ }j2\pi fct-t_0\}$

$y_1(t) = Re\{a_1sb(t-t_1)e\hat{\ }j2\pi fct-t_1\}$

$y_0(t) = Re\{a_2sb(t-t_2)e\hat{\ }j2\pi fct-t_2\}$…………………………

…..

…..

$yL-1(t)= Re\{aL-1sb(t-t_0)e\hat{\ }j2\pi fct-tL-1\}$

Net signal:-

$y(t)=\ Re\{\{\sum_{i=0}^{L-1} aisb(t-ti)e\hat{\ }-j2\pi fcti\}e\hat{\ }j2\pi fct\}$

Received complex baseband signal

$= yb(t) = \sum_{i=0}^{L-1} aisb(t-t0)e\hat{\ }-j2\,\pi fcti$

For a narrow band signal

sb(t-ti) = sb(t) (delay does not cause significant distortion)

So,

$yb(t) = sb(t)\sum_{i=0}^{L-1} aie\hat{\ }-j2\pi fcti$

The analytical model of the of a wireless system is given by

$yb(t)= h*sb(t)$

where h is complex fading coefficient given by $h =\sum_{i=0}^{L-1} aie\hat{\ }-j2\pi fcti$

2. Fading

The received signal power vexes and vein due to the constructive and destructive interference. The Figure 3 shows the power verses time when propagating the signal,

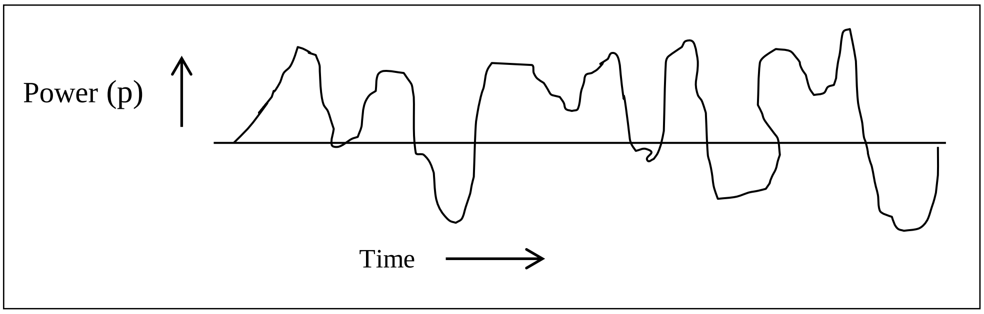


Figure 3. Power time distribution of a random signal.

This variation in power is known as fading which is due to the multipath propagation environment.

The analytical model of wireless system is given by:
yb(t) = h*sb(t)
where h is the complex fading system.

2.1 Statistics of fading system

H = $\sum_{i=0}^{L-1} aie^{\wedge} - j2\pi fcti$
This complex system can be represented as
h=x + jy =ae^jΦ
= $\sum_{i=0}^{L-1}(aicos2\pi fcti - jasin2\pi fcti)$
where x = $\sum_{i=0}^{L-1} aicos2\pi fcti$ and y = $\sum_{i=0}^{L-1} -aicos2\pi fcti$

The fading coefficient of a wireless system can be statistically represented as follows.
2.2 Complex fading coefficient
h=x+jy
where x and y are the sum of large number of random components

For x and y to be Gaussian in nature
x~N(0,1/2)
y~N(0,1/2)

Here x and y are independent random variables. A Gaussian random variable is defined as
X~N(C,σ^2)
f(x)=1/(√2πσ^2)$e^{-(x-\mu^2)/2\sigma^2}$ ………………………..(i)

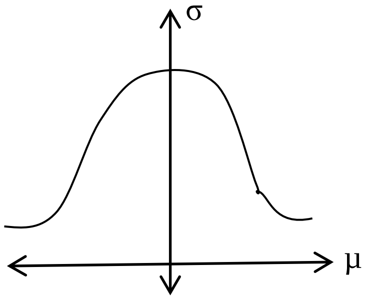


Figure4. Gaussian distribution.

For a standard Gaussian random variable,
μ =0 and σ^2= 1
f(x)= 1/$\sqrt{2}\pi$ $e^{-x^2/2}$…………………………………………(ii)

The equation is also known as probability density function of a standard gaussian distribution and it is depicted in Figure 4.

So x~N(0,1/2)

fx(x)= 1/$\sqrt{(2}\pi$*1/2) $e^{-\frac{x^2}{2*1/2}}$ = 1/$\sqrt{\pi} e^{-x^2}$
Similarly,
fy(y)= 1/$\sqrt{\pi} e^{-y^2}$

The joint distribution is given by
fx,y(x,y)= fx(x)*fy(y) = 1/ $\pi e^{-(x^2+y^2)}$ …………………….(iii)

Here, equation (iii) is representing a magnitude and phase
fA,Φ(a,Φ)= 1/ π*$e^{-(a^2\cos^2\Phi+y^2\sin^2\Phi)}$
= 1/ πe^{-a^2}

fa(A)= $\int_{-\pi}^{\pi} fA$,Φ(a, Φ)dΦ

Φ is the phase which varies from $-\pi$ to π

=$\int_{-\pi}^{\pi} a/\pi e^{-a^2}$

= 2ae^{-a^2} (where a = $\sqrt{x^2+y^2}$ is the envelope of the fading channel)

fa(A)=2ae^{-a^2} 0≤ $a \leq \infty$ …………………… (iv)

Here eqn (iv) is Rayleigh Distribution as depicted in Figure 5.

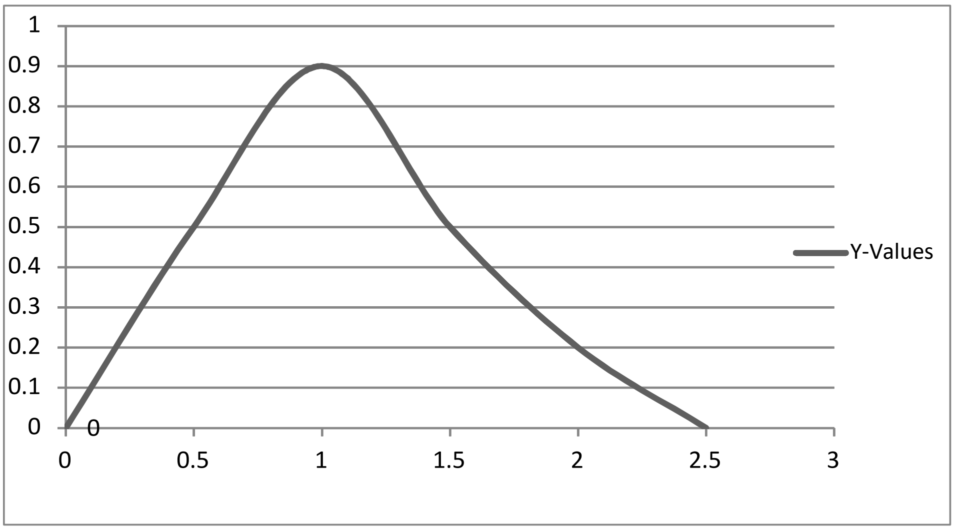


Figure5. Rayleigh distribution.

The uniform distribution of probability density function of Rayleigh distribution is given by
f(Φ)=$\int_0^\infty \frac{a}{\pi} 2ae^{-a^2} da$

f(Φ) $= 1/(2\,\pi)$

Uniform distribution of Rayleigh distribution

So,

yb(t)= h*sb(t)

Distribution of a = $2ae^{-a^2}$ $0 \leq a \leq \infty$
Density of Φ = $1/(2\,\pi)$ $-\pi \leq \Phi \leq \pi$

2.3 Deep fade

As we know the wireless communication system can be modeled as
y=h*x + n …………….(i)
where, y is received signal
x is transmitted signal
The received power is given by
Received power(Pr)=
where a is the magnitude of the random coefficient.
The performance of wireless coefficient is bad when received power is less than the noise power
Pr<Pn
<
< (
a < ……….. (ii)

This condition is known as deep fade event

The interference is destructive to an extent that almost very little signal power is received such that the magnitude of fading coefficient 'a' has to be less than $1/\sqrt{SNR}$. .This is known as deep fade event.

The probability density function of 'a' is the Rayleigh fading density. where a is a random quantity). So, the probability of a deep fade event is given by

P(a <)= ………….. (iii)

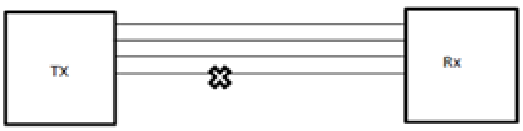
We know that, the probability that the random variable takes values in a range is simply the integral of the probability density function in that range.

So,
[If SNR is very high a= 0]
=
= 1/SNR

The probability of deep fade event in a wireless communication system is 1/SNR. The poor performance arises from "DEEP FADE". A deep fade is caused due to destructive interference. So, the multipath propagation environment has fundamentally altered the performance of the wireless communication system. The multipath propagation results in destructive interference because the signals from different path add up randomly in Amplitude and phase. Sometimes, it is constructive but sometimes it is destructive. This destructive interference results in deep fade. When the system is in deep fade, the bit error rate is very high. This causes the performance of wireless communication system to be very poor. So, it is a prominent question how to improve the performance of the wireless communication system.

3. Diversity

It is a technique that can be employed to improve the performance of the wireless communication system (for 3G and 4G). Diversity can be employed to improve performance of wireless communication system by controlling or combating the fading environment. Symbolically it is representing in Figure 6.



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Represents the fading link

Figure 6. Diversity

In this technique, we have diverse number of links over which information can be transmitted from Tx to Rx, so that even if 1 or 2 links are in deep fade the rest of the links carry the information.

To realize the diversity condition we need multiple antenna system.

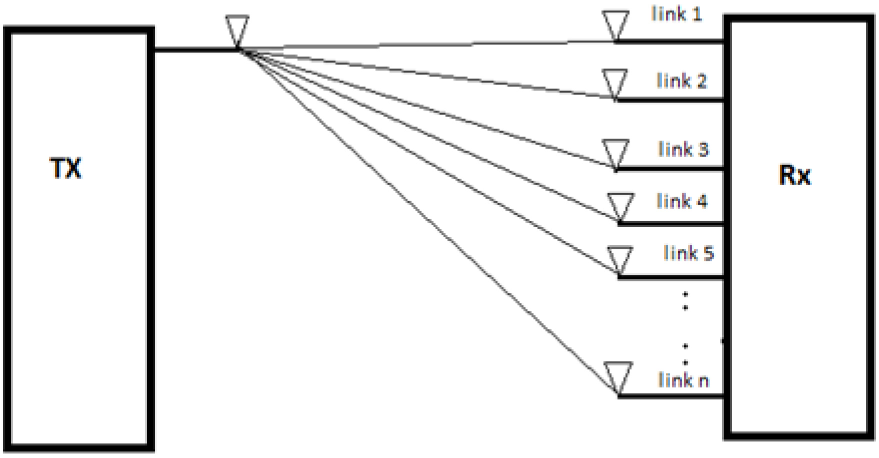

Figure 7. Multiple antenna system at receiver

In the Figure 7, link 1 and l4 are in deep fade. We can still receive the information over the link l2 and l3. Since we are increasing the number of links we have multiple copy. So, even if 1 or 2 copies of links get corrupted the rest of the copies can be used to receive or detect the transmitter information.

3.1 System model of multiple antennas

In a wireless system

y1 = h1x + n1

y2 = h2x + n2

.

.

yl= hlx + nl

It is the model of wireless communication having L receive antennas

$$\begin{bmatrix} y1 \\ y2 \\ \\ yl \end{bmatrix} = \begin{bmatrix} h1 \\ h2 \\ \\ hl \end{bmatrix} x + \begin{bmatrix} n1 \\ n2 \\ \\ nl \end{bmatrix}$$

3.2 Analysis of Rx antenna diversity system

The vector notation of above matrix is given as follows which is also the system model

$$\overline{y'} = h'x + \bar{n}$$

Here $\overline{y'}$ is the l dimensional receive vector, h' is the l dimensional channel coefficient vector and similarly is n' dimensional noise vector.

The Expected value of noise i.e the noise variance or the power of the noise at each receive antenna is

E{|hi(k)|^2}= σn^2 (considering the symmetric receive antenna)

3.2Signal Detection with maximal ratio combining

$y1, y2, \ldots\ldots\ldots . yl$ are the signals at l receive antenna. Let us combine these received signals to detect the transmitted signal antenna. Here we are weighing each signal at receive antenna by $w1^* \quad w2^* \ldots\ldots\ldots wl^*$ respectively. So we can represent in matrix form succinctly as follows

$$[w1^* \quad w2^* \ldots\ldots\ldots wl^*]\begin{bmatrix} y1 \\ y2 \\ \\ \\ yl \end{bmatrix} = w'^H y'(Beamforming\,)$$

$w' = \begin{bmatrix} w1 \\ w2 \\ \\ \\ wl \end{bmatrix}$ $\quad w'^H = transpose\ of\ w\ and\ its\ conjugate$ also known as w' hermitian.

$w'^H = [\,w1^* \ \ w2^* \ldots\ldots\ldots wl^*]\ conjugate$

By combining the w' hermitian with y' we are technically doing the beamforming and w' is the beamforming vector. After combining the beamforming output is given as

$w'^H y'$ (substituting the value of y')

$= w'^H(h'x + n'\,) = \ w'^H h'x + w'^H n'$

The first component is the signal component and second one is the noise component.

SNR = $= \frac{Signal\ power}{Noise\ Power}$ where, signal power $=|w'^H n'|^2 * P$

Noise power can be calculated as follows:

Effective noise = $w'^H n'$. The output power is given as

$E\{|w'^H n'|^2\} = E\{(w'^H n')(w'^H n')^*\}$

(where E is the expected value: we are using the expected value here because the noise is a random quantity)

We know that,

$w'^H n' = w1^* n1 + w2^* n2 + \cdots \ldots\ldots + wl^* nl$

$(w'^H n')(w'^H n')^* = (w1^* n1 + w2^* n2 + \cdots . + wl^*) * (w1n1^* + w2n2^* + \cdots + wl * nl)$

$E\{\sum_{i=1}^{l} |wi|^2 |ni|^2 + \sum_i \sum_j wiwj^* ni^* nj \ (i \neq j)\}$

$E(ninj^*) = E(ni)(nj^*) = 0$

Noise power the output of the beam former $= E\{\sum |wi|^2 |ni|^2\} = \ \sum |wi|^2\ E\{ni\}^2$

$= \sigma n^2 \sum |wi|^2$ (substituting the value of E{|hi(k)|^2}= σn^2)

$= \sigma n^2 w'^H w'$

So, signal to noise power SNR at the output of the beam former

SNR (in terms of beam forming vector) = $\frac{|w^H h|^2 P}{\sigma n^2 w'^H w'}$

We must choose w' such that SNR maximizes

SNR = $\left(\frac{|w^H h|^2}{w'^H w'}\right) * \left(\frac{P}{\sigma n^2}\right)$

Optimal beam forming vector can be chosen as w' = kw' (where k is a constant)

SNR = $\frac{k^2|w^H h|^2}{k^2 w'^H w'} * \left(\frac{P}{\sigma n^2}\right)$

choose w' such that || w' ||^2 =1

$w'^H w' = 1$

SNR = $\left(\frac{|w^H h|^2}{1}\right) * \left(\frac{P}{\sigma n^2}\right)$

w' = ch'. So the optimal beam forming vector is antenna coefficient vector multiplied by some constant.

$c^2||h'||^2 = 1$

$c = 1/(||h||)$ where $||h||$ is the norm of the channel coefficient vector

The optimal Beam forming vector w' that maximizes the received SNR is given by

w'opt = $\frac{h'}{||h||}$

This technique is known as maximal ratio combining (MRC). So, w'opt is simply the scaled version of the channel coefficient vector. So in a sense it is matched to the channel coefficient vector h'. So it acts as a matched filter. So it is also known as spatial matched filter.

Now, let us further derive SNR,

SNR = $\left|\frac{h'^H}{||h||} * h'\right| * \left(\frac{P}{\sigma n^2}\right)$

SNR =$||h||\left(\frac{P}{\sigma n^2}\right)$ (since $h'^H * h' = ||h||$)

4. Analysis of BER of Multiple Antenna System

Rx (receiver) at MRC (maximal ratio combining)

SNR = $||h||\left(\frac{P}{\sigma n^2}\right)$ This can be also written as

= $(|h1|^2 + |h2|^2 + \cdots \ldots\ldots + |hl|^2) * \left(\frac{P}{\sigma n^2}\right)$

= g$\left(\frac{P}{\sigma n^2}\right)$

where g= $(|h1|^2 + |h2|^2 + \cdots \ldots\ldots + |hl|^2)$ is called CHI – squared and is the gain of the multiple antenna system

It is random variable with 2L degrees of freedom. The distribution of this gain 'g' is given as

$$F_{G(g)} = \frac{1}{(L-1)!} g^{L-1} e^{-g}$$

We know that received SNR, Rx SNR = g$\left(\frac{P}{\sigma n^2}\right)$

Hence the, Instantaneous BER = Q* $(\sqrt{g * SNR})$

where the Q function is defined as: Q(x) = $\frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-x^2/2}$dx

To get the average BER , Average BER = $\int_0^{\infty} Q(\sqrt{g * SNR}\ ) F_{G(g)}\, dg$

BER with L receive antennas after MRC combining is given as

$$= \left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1}(L+l-1)C\, l \left(\frac{1+\lambda}{2}\right)^l$$

where,

$$\lambda = \sqrt{\frac{SNR}{2+SNR}}$$

Let us simplify this for L=1

$$= \left(\frac{1-\lambda}{2}\right)^1 \sum_{l=0}^{0} lC\, 1 \left(\frac{1+\lambda}{2}\right)^l$$

$$= \left(\frac{1-\lambda}{2}\right)^1\ 0C0 \left(\frac{1+\lambda}{2}\right)^0\ = \frac{1-\lambda}{2} = \frac{1}{2}\sqrt{\frac{SNR}{2+SNR}}$$

The above given expression is the BER with single receive antenna in Rayleigh channel.

For High SNR:

$$= \frac{1}{2}(1-\lambda) = \frac{1}{2}\left(1-\sqrt{\frac{SNR}{2+SNR}}\right)$$

Using the binomial expansion, we get

$$\text{BER} = \frac{1}{2}\left(1-\frac{1}{\left(\frac{2}{SNR}+\frac{SNR}{SNR}\right)^{\frac{1}{2}}}\right) = \frac{1}{2}\left[1-\left(1-{}^1\!/_2 * \frac{2}{SNR}\right)\right] = \frac{1}{2SNR}$$

Also

$$\frac{1}{2}(1+\lambda) = \frac{1}{2}\left(1+1-\frac{1}{SNR}\right) = \frac{1}{2}(1+1) = 1$$

So at High SNR, λ is very small

$$\frac{1}{2}(1+\lambda) = 1$$
$$\frac{1}{2}(1-\lambda) = \frac{1}{2SNR}$$

Average BER =

$$\left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1}(L+l-1)C\, l \left(\frac{1+\lambda}{2}\right)^l = \left(\frac{1}{2SNR}\right)^L \sum_{l=0}^{L-1}(L+l-1)C\, l = \text{2L-1 CL}\ \frac{1}{2^{\wedge}L}\left(\frac{1}{SNR}\right)^{\wedge}L$$

Let us take an example to show how diversity reduces the power required.
L=2 receive antennas. What is the SNR required to achieve a bit error rate (BER) of 10^{-6}?

$$\text{BER} = \text{2L-1 CL}\ \frac{1}{2^{\wedge}L}\left(\frac{1}{SNR}\right)^{\wedge}L$$
$$10^{-6} = 3C_2 \frac{1}{2^{\wedge}2}\left(\frac{1}{SNR}\right)^{\wedge}2$$
$$\text{SNR} = \frac{\sqrt{3}}{2} * 10^3$$

SNR in dB = $10log_{10}(\frac{\sqrt{3}}{2} * 10^3)$

= 29.37dB

The SNR required with only one antenna to achieve 10^{-6} = 57 dB

So, the reduction in SNR = 57-29.37 dB = 28dB

If $P^1w = power\ required\ with\ 1\ Rx\ antenna$
$P^2\ w = power\ required\ with\ 2\ Rx\ antenna$
$10log_{10}(\frac{P^1w}{P^2\ w})$ = 28 dB $\approx 30dB$
$(P^1w\ )/(P^2\ w)$ = 1000
$P^2\ w = \frac{P^1w}{1000}$ $\therefore$ P^2w is thousand times lower than P^1w

Adding one more antenna at the Rx side has significantly lowered the power required to transmit. Hence receive diversity is very important in 3G/4G. It is also employed in WCDMA, HSDPA, LTE, Wi-MAX. All these systems use receive as well as transmit diversity.

4.1 Relationship between Bit Error Rate and Receive Antennas

BER = 2L-1 CL $\frac{1}{2^\wedge L}(\frac{1}{SNR})^\wedge L$
L=1, BER $\propto \frac{1}{SNR}$
L=2, BER $\propto \frac{1}{SNR^2}$
L=3, BER $\propto \frac{1}{SNR^3}$
As the no of Rx antennas is increasing the exponent of the SNR in denominator is increasing. Hence the BER is decreasing at much faster rate.

4. Conclusion

It is confirmed that the performance of a wireless communication system is measured in terms of BER. The introduction of multiple antennas at the receiver end can reduce the BER at faster rate as it decreases with the additional antenna. The same method can be applied at transmitter end also to enhance the performance by reducing the BER. This concept of using multiple antennas at both sides i.e. Transmitter and Receiver side is considered as multiple input and multiple output (MIMO) system; it is widely being used in today’s wireless communication system for the better performance in 3G, LTE and 4G mobile communication system.

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